

西方经济学

Part 3 Growth Theory

Lecture 3 The Solow–Swan Model

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Supplement Readings

西方经济学

- (1) M16; S9.¹
- (2) 其他文献: Loyalka, P. et al. 2021. [Skill levels and gains in university STEM education](#) in China, India, Russia and the United States. *Nature Human Behaviour* 5, 892–904.
- (3) 其他文献: CAPITAL ACCUMULATION AS A SOURCE OF GROWTH
- (4) 其他文献: POPULATION GROWTH AND TECHNOLOGICAL PROGRESS
- (5) 其他文献: GROWTH EMPIRICS AND POLICY



¹M 指代马工程教材, S 指代课外阅读材料沈坤荣教程。

学习目标

西方经济学

- (1) 理解长期增长的动力。
- (2) 掌握稳态概念，分析稳态的改变。
- (3) 掌握收敛假说与收敛速度。
- (4) 了解增长核算的 Solow 残差法。
- (5) 掌握马工程教材精神。



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观察与思考 I

中国经济增长的动力是什么？

- (1) **人民日报评论** (2024 年 03 月 12 日第 5 版): 新质生产力 (new productive forces) 是**创新**起主导作用, 具有高科技、高效能、高质量特征, 符合新发展理念的先进生产力质态。它由技术革命性突破、生产要素创新性配置、产业深度转型升级而催生, 以劳动者、劳动资料、劳动对象及其优化组合的跃升为基本内涵, 以**全要素生产率大幅提升**为核心标志, 特点是创新, 关键在质优, 本质是先进生产力。因此, 形成新质生产力的过程也是战略性新兴产业加快发展、未来产业孕育壮大的过程。新质生产力已经在实践中形成并展示出对高质量发展的强劲推动力、支撑力。**人形机器人**不仅能浇花、洗碗、摆放椅子, 甚至能帮人剃胡须; **量子计算机**能把算力提高到新的层次, 以指数级增长超越现在的超级计算机……这些前沿创新让我们切实认识到, 科技创新能够催生新产业、新模式、新动能, 是发展新质生产力的核心要素。推动新质生产力加快发展, 必须加强科技创新特别是原创性、颠覆性科技创新, 加快实现高水平科技自立自强, 打好关键核心技术攻坚战, 使原创性、颠覆性科技创新成果竞相涌现。同时, 必须坚持全面深化改革, 推动形成与之相适应的新型生产关系。
- (2) **新华网** (2023 年 2 月 28 日): 中国经济为什么行——世界看多中国的“四力”逻辑。1. 政策端的统筹支撑力; 2. 需求端的消费驱动力; 3. 供给端的产业竞争力; 4. 联动世界的开放融合力。

观察与思考 II

中国经济增长的动力是什么？

- (3) **人民智库专家** (2021 年 04 月 26 日): 改革开放以来的经济增长分为三个阶段。第一阶段是**生产要素粗放驱动**的阶段。由于对生产要素的利用严重不足, 短缺与失业并存, 制度和结构性改革可以促使大规模生产要素投入的增长。第二阶段是**生产要素配置效率提高**的阶段。随着中国经济的进一步发展, 要素投入增量逐渐消失, 要素配置效率的提高成为经济增长的新动力, 而生产要素配置效率提高的关键是要素市场的发育和健全。第三阶段是**创新驱动**的阶段。随着要素市场的改革和完善, 要素配置效率得以保证, 这时, 战略性科技突破将成为新增长动力的核心。

目前, 中国经济增长的**第一阶段显然已经结束**。随着本国经济的发展和壮大, 中国面临的全球竞争空前激烈, 加之地缘政治与全球治理结构的变化, 可能没有时间让我们依次经历第二阶段和第三阶段, 而必须是熔第二阶段与第三阶段于一炉, 即**同时进入要素配置效率提高阶段和创新驱动阶段**。

- (4) 邓小平 (1988 年 9 月 5 日会见捷克斯洛伐克总统胡萨克时的谈话, 《邓小平文选》第三卷, p.274-275): 马克思说过, 科学技术是生产力, 事实证明这话讲得很对。依我看, **科学技术是第一生产力**。……**把教育问题解决好**。



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- (1) 中国人民银行《深入推进利率市场化改革》(2022年9月20日): 利率是资金的价格, 是重要的宏观经济变量, 决定着资金的流向, 对宏观经济均衡和资源配置有重要导向意义。理论上, 自然利率是宏观经济总供求达到均衡时的真实利率水平。中长期看, 宏观意义上的真实利率水平应与自然利率基本匹配。实践中一般采用“黄金法则”来衡量合理的利率水平, 即经通胀调整后的**真实利率 r 应与实际经济增长率 g 大体相等**。若真实利率持续高于潜在增速, 会导致社会融资成本高企, 企业经营困难, 不利于经济发展。真实利率低于潜在经济增速, 有利于债务可持续, 可以给政府一些额外的政策空间。但如果真实利率持续明显低于潜在经济增速, 会扭曲金融资源配置, 带来过度投资、产能过剩、通货膨胀、资产价格泡沫、资金空转等问题, 难以长期持续。我国货币政策始终坚持以我为主, 以静制动, 引导市场利率水平稳中有降, 效果较好。目前我国定期存款利率约为 1% 至 2%, 贷款利率约为 4% 至 5%, **真实利率略低于潜在实际经济增速, 处于较为合理水平**, 是留有空间的最优策略。当前我国的经济增长、物价水平、就业状况、国际收支平衡等货币政策调控目标均运行在合理区间, 从实际效果上也充分验证了我国当前的利率水平总体上处于合理区间。

$$r \lesssim \frac{Y}{Y}$$



Outline

- 1 Definition of Economic Growth
- 2 Capital Accumulation
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马工程观点 (M, 2019, p.249)

一、经济增长和经济发展

从本书第九章中可知，在宏观经济学中，国内生产总值（GDP）既是衡量一个国家（或地区）经济活动的重要指标，也是反映该国（或该地区）在一定时期内生产总成果的重要指标。因此，从理论的层面看，为了描述和反映一个经济体（国家或地区）物质产品的丰富和增加，很自然地联系到以 GDP 表示的产量的概念。

一般地，在宏观经济学中，经济增长被定义为产量的增加，这里，产量既可以表示为经济的总产量（GDP 总量），也可以表示为人均产量（人均 GDP）。经济增长的程度可以用增长率来描述。

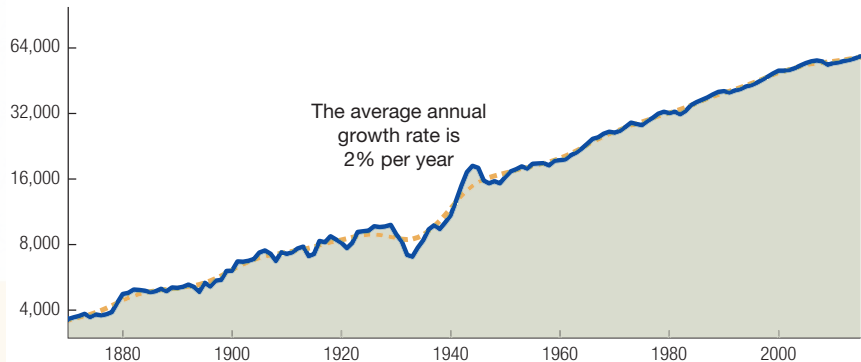
实际上是对经济波动的描述。

波动

Jones's (2020, p.12) Macroeconomics, 5th ed.

Per Capita GDP in the United States

Per capita GDP
(2017 dollars)



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Capital Accumulation

in the Solow (1956)–Swan (1956) Model

Assumption 1

The production function is neoclassical.

Assumption 2

L and A are constant.

Let $y = Y/L$, $k = K/L$.

定义: $\frac{df}{dk} = f'(k)$

$$Y = F(K, L) \Rightarrow y = f(k)$$

Saving = Investment

With the assumption of $S = I$, the fundamental equation is

$$\dot{K}(t) = s \cdot Y(t) - \delta \cdot K(t), \quad \forall t \geq 0 \text{ or}$$

$$\dot{k} = s \cdot f(k) - \delta \cdot k, \quad \forall t \geq 0$$

$f'(k) > 0, f''(k) < 0$



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$$\begin{array}{cc} | & | \\ K_t & K_{t+1} \end{array}$$

$$K_{t+1} = \underbrace{K_t + I_t}_{GI} - \delta K_t$$

$$K_{t+1} - K_t = I_t - \delta K_t$$

$$\dot{K} = \frac{dK}{dt} = \lim_{t \rightarrow \infty} K_{t+1} - K_t = I(t) - \delta K(t) = s \cdot Y(t) - \delta K(t).$$

$$I(t) = S(t) = s \cdot Y(t) \quad s \in (0, 1)$$

$$\dot{K} = s \cdot Y - \delta K$$

$$k = \frac{K}{L} \Rightarrow \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{s \cdot Y/L}{K/L} - \delta - \frac{\dot{L}}{L}$$

$$= s \frac{f(k)}{k} - \delta - n$$

$$\dot{k} = s \cdot f(k) - (\delta + n)k$$

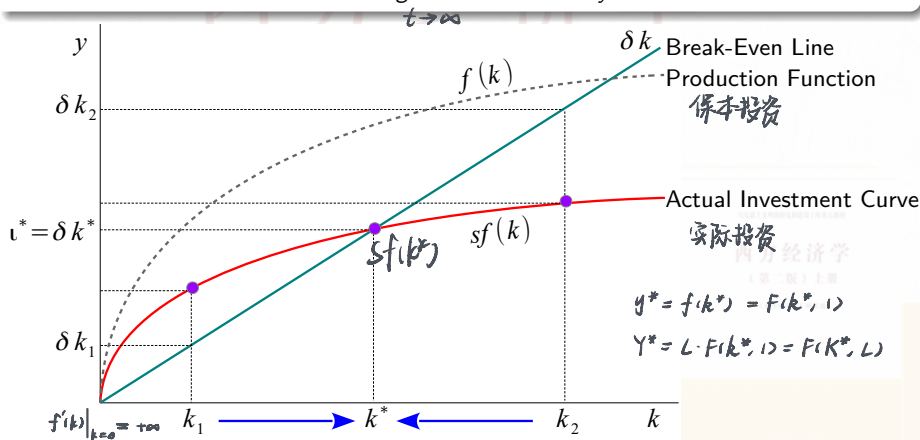
actual investment

break-even investment

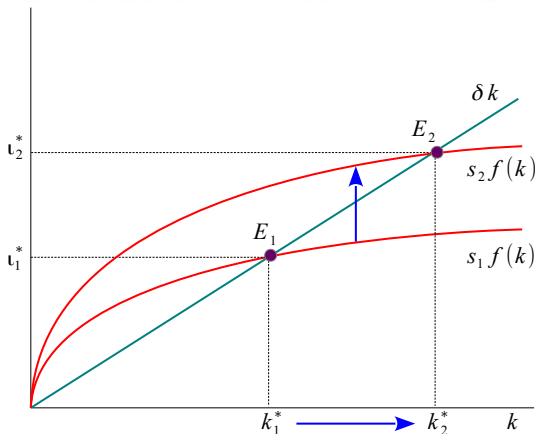
Balanced Growth Path

Definition 1 均衡增长路径

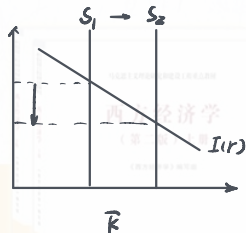
A *balanced growth path* is defined as a situation in which various quantities grow at constant rates. It is also called the *long run state* or *steady state*.



Effects of Raising Saving Rate s



$$\begin{aligned} \dot{k} &= s f(k) - \delta k = 0 \\ \Rightarrow s f(k^*) &= \delta k^* \\ f(k^*) + s f'(k^*) \frac{\partial k^*}{\partial s} &= \delta \frac{\partial k^*}{\partial s} \\ f(k^*) &= \frac{\partial k^*}{\partial s} (\delta - s f'(k^*)) \\ \frac{\partial k^*}{\partial s} &= \frac{f(k^*)}{\delta - s f'(k^*)} > 0 \end{aligned}$$



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Golden Rule Steady State 黄金法则稳态

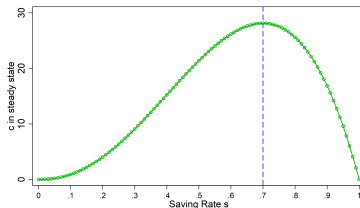
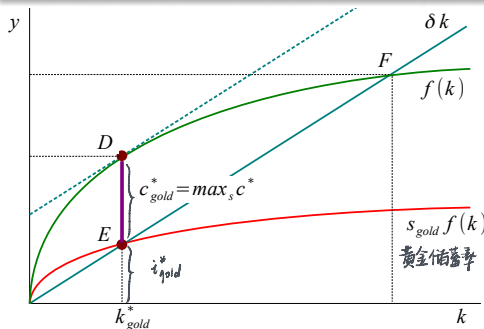
$$\begin{aligned} \dot{k} &= 0 : sf(k^*) = \delta k^* \\ c^* &= f(k^*) - sf(k^*) \\ &= f(k^*) - \delta k^* \end{aligned}$$

Definition 2

$$c_{gold}^* = \max_s c^* : \frac{\partial c^*}{\partial s} = (f'(k^*) - \delta) \frac{\partial k^*}{\partial s} = 0$$

The policymaker chooses the saving rate s such that the steady state value of c is maximized. The optimal saving rate is called the golden rule level of the saving rate, and is denoted by s_{gold} . The steady state value of capital corresponding to s_{gold} is called the golden rule level of capital, and is denoted by k_{gold}^* .

$$\text{有 } f'(k_{gold}^*) = \delta.$$

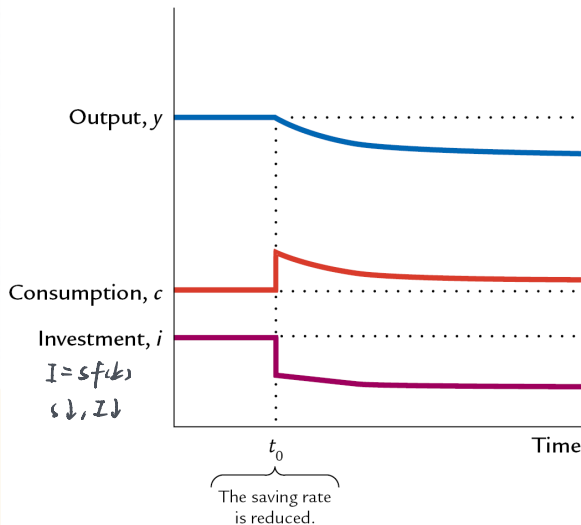


Note: depreciation rate = .1; production function is given by $\ln(y) = a \ln(k)$, where $a = .7$.

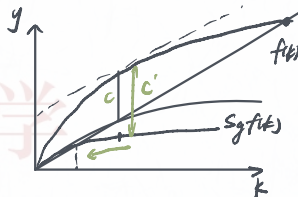
Golden Rule Level of Consumption per Capital

The Transition to the Golden Rule Steady State

Starting with Too Much Capital



Mankiw, Macroeconomics, 11e, © 2022 Worth Publishers



最初的稳态资本水平为 k^* 。
 在 t_0 时刻, $s \downarrow, i \downarrow, c \uparrow$ 。资本存量仍为 k^* 。
 故 c, I 跳跃, y 不跳跃。
 $s \downarrow, i \downarrow, \bar{i} < \delta(k)$, $k^* \downarrow \Rightarrow y \downarrow, c \downarrow, i \downarrow$

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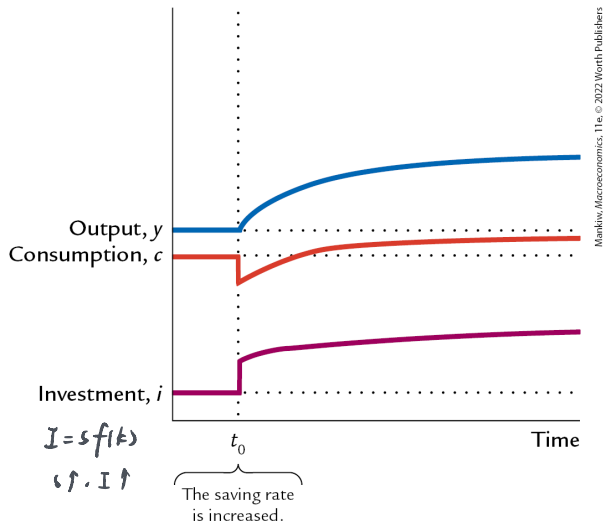
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The Transition to the Golden Rule Steady State

Starting with Too Little Capital



同上。

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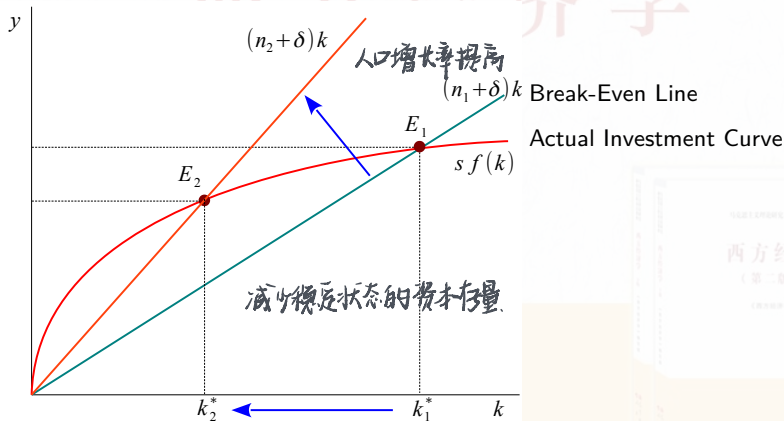
Effects of Raising Population Growth n

Suppose $\dot{L}/L = n$. The fundamental equation is

$$\dot{k} = sf(k) - (n + \delta)k$$

忽略了人的创造性?

$n \uparrow \Rightarrow k^* \downarrow \Rightarrow y^* \downarrow$



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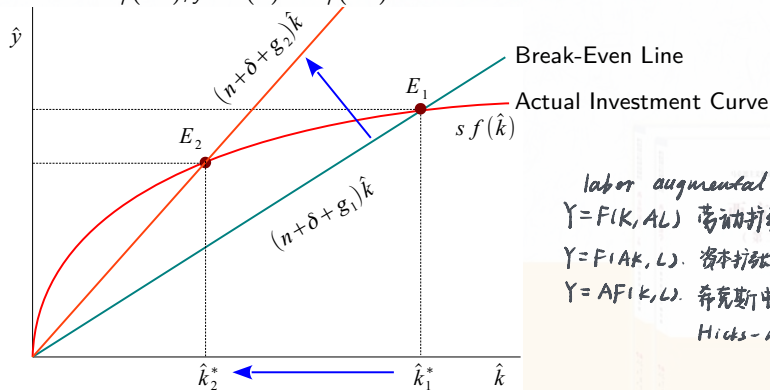
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Technological Progress 科技进步

Suppose $\dot{A}/A = g$. The production function is $Y = F(K, AL)$ where AL is called the effective labor/worker.² The fundamental equation is

$$\hat{k} = sf(\hat{k}) - (n + \delta + g)\hat{k}$$

where $\hat{k} = K/(AL)$, $\hat{y} = f(\hat{k}) = Y/(AL)$.



labor augmental progress
 $Y = F(K, AL)$ 劳动扩张型, 劳动生产率↑.
 $Y = F(AK, L)$ 资本扩张型.
 $Y = AF(K, L)$ 希克斯中性
 Hicks-neutral

² Effective: Actual. // Effective demand. (Merriam-Webster).

$$\dot{K} = sY - \delta K$$

$$\dot{K} = sf(k) - (n + \delta)k$$

$$Y = F(K, AL) \xRightarrow{\text{有效劳动}} \frac{Y}{AL} = F\left(\frac{K}{AL}, \frac{AL}{AL}\right) \Rightarrow \hat{y} = F(\hat{k}, 1) = f(\hat{k})$$

$$f'(\hat{k}) = F'(\hat{k}, 1) \quad \hat{k} = \frac{K}{AL} \quad A(t) = A(0)e^{gt} \quad L(t) = L(0)e^{nt}$$

$$\frac{\dot{\hat{k}}}{\hat{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L} = \frac{sY/AL}{K/AL} - (\delta + g + n) = s \frac{f(\hat{k})}{\hat{k}} - (\delta + g + n)$$

$$\dot{\hat{k}} = sf(\hat{k}) - (\delta + g + n)\hat{k}$$

$$\text{由 } \dot{\hat{k}} = 0, \text{ 有 } sf(\hat{k}^*) = (\delta + g + n)\hat{k}^*.$$

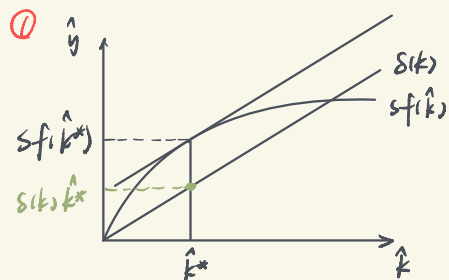
$$\frac{f(\hat{k}^*)}{\hat{k}^*} = \frac{\delta + g + n}{s}$$

① 对 s 求偏导, 有

$$\frac{f'(\hat{k}^*)\hat{k}^* - f(\hat{k}^*)}{(\hat{k}^*)^2} \frac{\partial \hat{k}^*}{\partial s} = \frac{-(\delta + g + n)}{s^2} \quad \frac{\partial \hat{k}^*}{\partial s} > 0.$$

$$f(\hat{k}) = F(\hat{k}, 1)$$

$$Y = F(K, AL) \Rightarrow Y = AL \cdot F(\hat{k}, 1).$$



$f'(\hat{k}^*)\hat{k}^* - f(\hat{k}^*)$ 为什么小于 0.

乘 s $s f'(\hat{k}^*)\hat{k}^* - s f(\hat{k}^*)$

$s f'(\hat{k}^*) = \delta(k)$ 斜率相等.

$$\delta(k)\hat{k}^* < s f(\hat{k}^*)$$

$$\Rightarrow \delta(k)\hat{k}^* - s f(\hat{k}^*) < 0$$

$$f'(\hat{k}^*)\hat{k}^* - f(\hat{k}^*) < 0.$$

② $Y = F(k, AL) \Rightarrow Y = AL \cdot F(\hat{k}, 1)$

$$Y'_L = A F(\hat{k}, 1) - AL F'_1(\hat{k}, 1) \frac{k}{AL^2}$$

$$= A(f(\hat{k}) - f'(\hat{k})\hat{k}) > 0$$

由于 $\hat{k} + 1$,
存在 $s, \delta(k)$. $\hat{y} + 1$
故 $f(k) \uparrow$

$f'(\hat{k}) < 1$

即可构造 $\frac{f(\hat{k})}{\hat{k}}$

$$f'(\hat{k}) = \frac{\hat{k} f'(\hat{k}) - f(\hat{k})}{\hat{k}^2} < 0 \quad \text{证} \quad \frac{f(\hat{k})}{\hat{k}} \text{ 递减. 即证.}$$

对 L 求偏导, 有 $Y_L' = A F_L(\hat{k}, 1) - A L F_{LL}(\hat{k}, 1) \frac{K}{AL}$.

$$\textcircled{E} = A[f(\hat{k}) - f'(\hat{k})\hat{k}] > 0$$

$$sf(k^*) = (\delta + g + n)k^*.$$

$$\hat{c}^* = \frac{C^*}{AL} = f(\hat{k}^*) - sf(k^*) = f(\hat{k}^*) - (\delta + g + n)k^*.$$

$$\frac{\partial \hat{c}^*}{\partial s} = [f'(\hat{k}^*) - (\delta + g + n)] \frac{\partial k^*}{\partial s} = 0$$

$$r^* = f'(\hat{k}^*) - \delta = g + n = \frac{\dot{Y}}{Y}.$$

Real rental rate.

$$MP_K = \frac{R}{P}.$$

$$Y = F(XK, AL),$$

$$A = A_0 e^{gt}, L = L_0 e^{nt}, X = X_0 e^{xt}, K = K_0 e^{\gamma t}.$$

$$\dot{K} = sY - \delta K.$$

F is neo-classical.

If the economy has a steady state, then F can be written as $Y = \tilde{F}(K, BL)$.

$$\frac{\dot{K}}{K} = \bar{s} \cdot \frac{Y}{K} - \bar{\delta} \Rightarrow \frac{\bar{Y}}{K}$$

$$Y = F(XK, AL) \Rightarrow \frac{Y}{K} = F(X, \frac{AL}{K}) = X \cdot F(1, \frac{AL}{XK}).$$

$$u \triangleq \frac{AL}{XK}, A = A_0 e^{gt}, L = L_0 e^{nt}, X = X_0 e^{xt}, K = K_0 e^{\gamma t}.$$

$$u = \frac{AL}{XK} = \frac{A_0 L_0}{X_0 K_0} e^{(g+n-x-\gamma)t}. \quad u \triangleq u_0 e^{(g+n-x-\gamma)t} \quad F(\frac{AL}{XK}) \triangleq f(u_0 e^{(g+n-x-\gamma)t})$$

$$\frac{Y}{K} = X_0 e^{xt} f(u_0 e^{(g+n-x-\gamma)t}) \Rightarrow (\frac{Y}{K})' = (X_0 e^{xt} f(u_0 e^{(g+n-x-\gamma)t}))'.$$

$$0 = X_0 (x e^{xt} f(u) + e^{xt} f'(u) u_0 (g+n-x-\gamma) e^{(g+n-x-\gamma)t}) \Rightarrow -x f(u) = f'(u) u (g+n-x-\gamma).$$

$$if (g+n-x-\gamma) = 0 \Rightarrow x = 0 \Rightarrow X = Constant. \quad B \triangleq \frac{A}{X} \Rightarrow Y = \tilde{F}(K, BL).$$

$$if (g+n-x-\gamma) \neq 0 \Rightarrow \frac{1}{u} \frac{x}{x+\gamma-g-n} = \frac{f'(u)}{f(u)}. \quad \beta \triangleq \frac{x+\gamma-g-u}{x}$$

$$\int_{u_0}^u \frac{f'(u)}{f(u)} du = \int_{u_0}^u \beta \frac{1}{u} du \Rightarrow \ln f(u) - \ln f(u_0) = \beta f(\ln u - \ln u_0) \Rightarrow \ln \frac{f(u)}{f(u_0)} = \ln \left(\frac{u}{u_0} \right)^\beta$$

$$\Rightarrow \frac{f(u)}{f(u_0)} = \left(\frac{u}{u_0} \right)^\beta \Rightarrow f(u) = \left(\frac{u}{u_0} \right)^\beta f(u_0). \quad u \triangleq \frac{AL}{XK}$$

$$\beta \triangleq \frac{x + \gamma - g - n}{x} \quad \alpha \triangleq \frac{\gamma - g - n}{x} \quad \Rightarrow \beta = 1 - \frac{\gamma - g - n}{x} = 1 - \alpha.$$

$$\Rightarrow f(u) = \frac{f(u_0)}{u_0^\beta} \left(\frac{AL}{XK} \right)^\beta = \frac{f(u_0)}{u_0^\beta} K^{-\beta} \left(\frac{AL}{X} \right)^\beta. \quad C \triangleq \frac{f(u_0)}{u_0^\beta}$$

$$Y = XK \cdot F(1, u) = XK \cdot C \cdot \left(\frac{AL}{XK} \right)^\beta = C(XK)^{1-\beta} (AC)^\beta = CK^{(1-\beta)} (AX^{(\frac{1-\beta}{\beta})} L)^\beta.$$

$$B \triangleq AX^{\frac{1-\beta}{\beta}} \Rightarrow Y \triangleq \tilde{F}(K, BL).$$

马工程观点 (M, 2019, p.265)

表 16-4 具有技术进步的新古典增长模型的稳态增长率

内生变量	符号	稳态增长率
按有效劳动平均的资本	$\hat{k} = \frac{K}{AN}$ $\hat{k} = \frac{k}{AL}$	$\frac{\dot{\hat{k}}}{\hat{k}} = 0$ 0
按有效劳动平均的产量	$\hat{y} = \frac{Y}{AN}$ $\hat{y} = \frac{y}{AL}$	$\frac{\dot{\hat{y}}}{\hat{y}} = 0$ 0 $g+n+\delta$
人均资本	$\frac{K}{N} = \hat{k}A$ $k = \frac{k}{L}$	$\frac{\dot{k}}{k} = g$ g_A
人均产量	$\frac{Y}{N} = \hat{y}A$ $y = \frac{y}{L}$	$\frac{\dot{y}}{y} = g$ g_A
总资本	$K = \hat{k}AN$	$\frac{\dot{K}}{K} = g+n$ $g_N + g_A$
总产量	$Y = \hat{y}AN$	$\frac{\dot{Y}}{Y} = g+n$ $g_N + g_A$



The Golden Rule

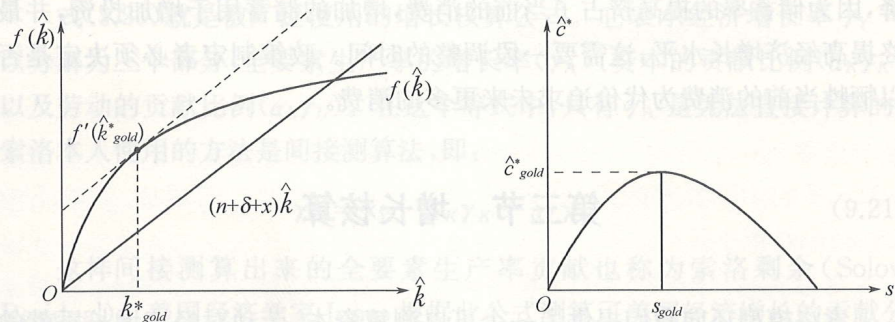


图 9-13 最优储蓄率的确定

Example 1 (观察与思考)

如何理解“实际利率的黄金法则”？



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Sources of A ↑: Intentional Technological Change

Joseph A. Schumpeter (1942, p.83), *Capitalism, Socialism and Democracy*:

*“The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumers’ goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates.[This process] incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one. This process of **Creative Destruction** is the essential fact about capitalism.”*

创造性破坏



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Outline

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Convergence in the Solow Model 索罗模型中的收敛.

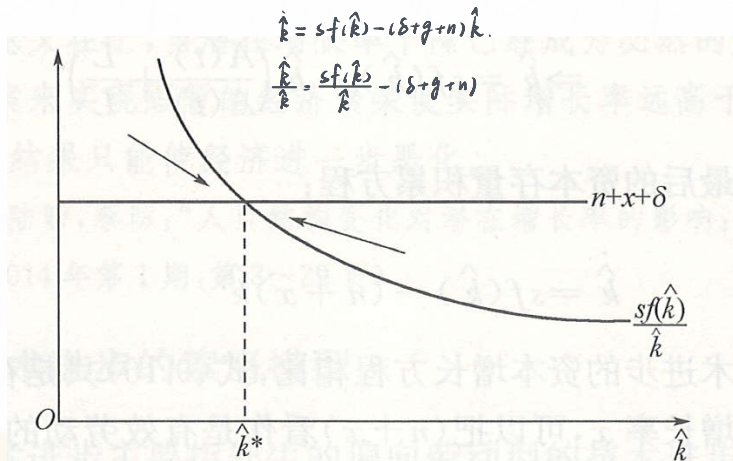


图 9-11 考虑技术进步的稳态



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Conditional Convergence

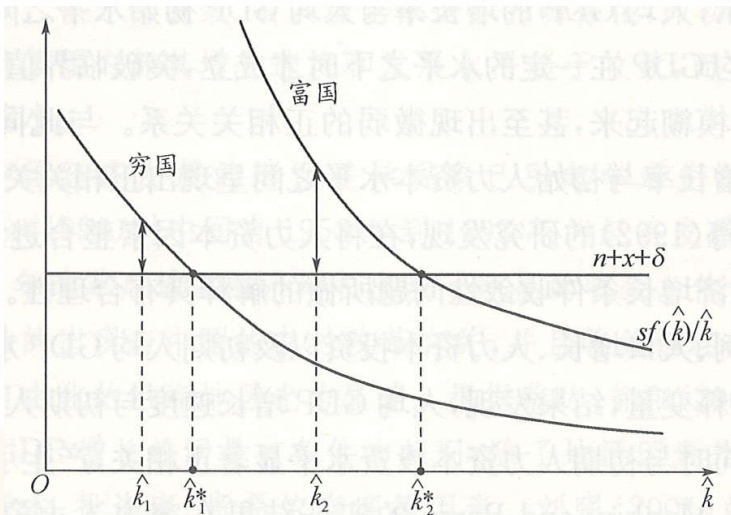


图 9-16 不同国家的经济增长率

The Speed of Convergence 收敛的速度

$$\dot{\alpha} = -\lambda \cdot \alpha(t)$$

$$-\frac{\partial \alpha}{\partial \alpha} = \lambda = -\frac{\dot{\alpha}}{\alpha}$$

The speed of convergence λ is measured by how much the capital growth rate declines as the capital stock increases in a proportional sense. It is given by $\lambda(\hat{k}) = -\frac{d(\hat{k}/\hat{k})}{d \ln \hat{k}}$. In the neighborhood of $\hat{k}^*$,

$$\begin{aligned} \frac{\dot{\hat{k}}}{\hat{k}} &= \frac{sf(\hat{k})}{\hat{k}} - (n + g + \delta) \\ &= -(1 - \alpha_K^*)(n + \delta + g) \cdot (\ln \hat{k} - \ln \hat{k}^*) + o(\ln \hat{k} - \ln \hat{k}^*) \\ &= -\lambda^* \cdot (\ln \hat{k} - \ln \hat{k}^*) + o(\ln \hat{k} - \ln \hat{k}^*), \end{aligned}$$

资本价格 × 人均有效资本

where $\alpha_K^* = \frac{f'(\hat{k}^*)\hat{k}^*}{f(\hat{k}^*)}$ and $\lambda^* = (1 - \alpha_K^*)(n + \delta + g)$. Intuitively, λ^* can be interpreted as

资本增长比例

$$\lambda^* \approx \frac{\Delta \ln \hat{k}}{(\ln \hat{k}^* - \ln \hat{k}) \Delta t} \cdot \frac{\text{增长率}}{\text{总量}} = \text{增长比例}$$



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$$\frac{\dot{\hat{k}}}{\hat{k}} = \frac{sf(\hat{k})}{\hat{k}} - (\delta + g + n) = \frac{d \ln \hat{k}}{dt} = \dot{x} \quad x = \ln \hat{k} \Rightarrow \hat{k} = e^x$$

$$\dot{x} = \frac{sf(e^x)}{e^x} - (\delta + g + n) = h(x) \quad x^* = \ln \hat{k}^*$$

$$\hat{k} = \frac{K}{AL}$$

1 Definition of Economic Growth

$$h(x) = h(x^*) + h'(x^*)(x - x^*) + o(x^*)$$

2 Capital Accumulation

$$= s \frac{f(e^x) \cdot e^x - f(e^x^*) \cdot e^{x^*}}{(e^x)^2} (x - x^*) + o(x^*)$$

3 Population Growth

$$= s \frac{f'(\hat{k}^*) \cdot \hat{k}^* - f(\hat{k}^*)}{\hat{k}^*} (x - x^*) + o(x^*)$$

4 Technological Progress

$$= \frac{sf(\hat{k}^*)}{\hat{k}^*} \left[\frac{\hat{k}^* f'(\hat{k}^*)}{f(\hat{k}^*)} - 1 \right] (x - x^*) + o(x^*)$$

6 Growth Accounting

$$= -(\delta + g + n)(1 - \alpha_k)(x - x^*) + o(x^*)$$

$$\dot{x} \doteq -\lambda(x - x^*) \quad \lambda = (1 - \alpha_k)(\delta + g + n)$$

7 Policies to Promote Growth

8 马工程教材疑难重点



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Growth Accounting

全要素生产率

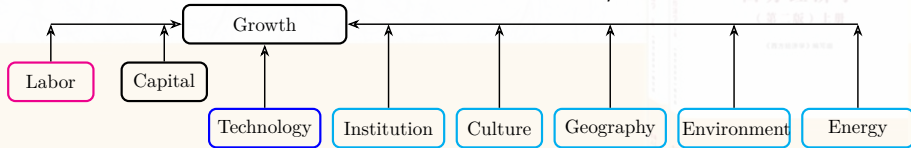
Suppose the production function is $Y = F(K, L, A)$. In growth accounting, A is often called the **total factor productivity (TFP)** which captures effects on output level caused by factors other than K and L .

$$\frac{\dot{Y}}{Y} = \frac{KF_K}{Y} \frac{\dot{K}}{K} + \frac{LF_L}{Y} \frac{\dot{L}}{L} + \frac{AF_A}{Y} \frac{\dot{A}}{A} \Rightarrow \gamma_Y = \alpha_K \gamma_K + \beta_L \gamma_L + \chi_{TFP}$$

Let $\alpha_K \triangleq \frac{KF_K}{Y}$, $\beta_L \triangleq \frac{LF_L}{Y}$ denote the factor income shares. Let $\chi_{TFP} \triangleq \frac{AF_A}{Y} \frac{\dot{A}}{A}$ denote the contribution of TFP.

索洛剩余法计算 TFP: $\chi_{TFP} = g_Y - \alpha_K g_K - \beta_L g_L$.

If F is homogeneous of degree one in K and L , then $\alpha_K + \beta_L = 1$.



total factor productivity (TFP)

The Slowdown in Productivity Growth 生产率下降

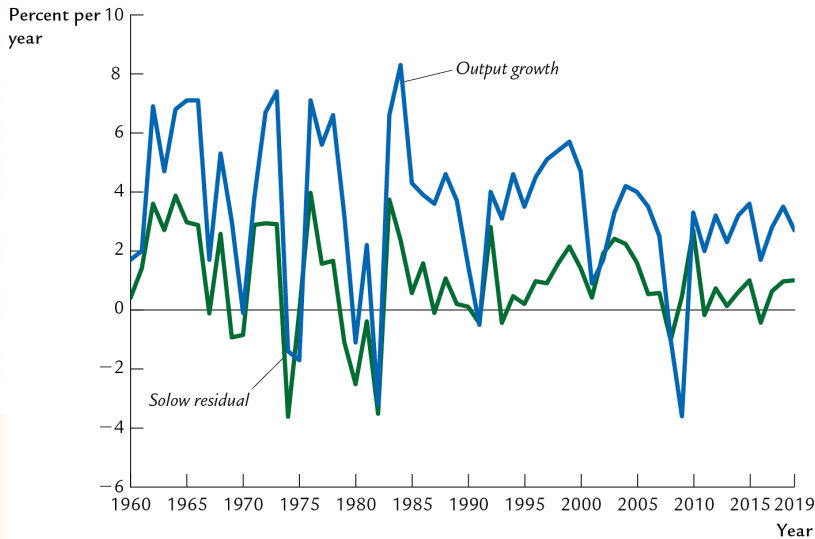
TABLE 10-1 Accounting for Economic Growth in the United States

SOURCES OF GROWTH					
Years	Output Growth $\Delta Y/Y$	=	Capital $\alpha \Delta K/K$	+ Labor $(1-\alpha)\Delta L/L$	+ Total Factor Productivity $\Delta A/A$
(average percentage increase per year)					
1948–2019	3.4		1.3	1.0	1.1
1948–1973	4.2		1.3	1.0	1.9
1973–2019	3.0		1.3	1.1	0.7

Data from: U.S. Department of Labor. Data are for the nonfarm business sector. Parts may not add to total due to rounding.



Growth in Output and the Solow Residual 正相关



Mankiw, Macroeconomics, 11e, © 2022 Worth Publishers



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Growth Accounting for China

余淼杰、于鸿君 (2022), 《宏观经济学原理——中国视角》

表 3-3 中国经济增长核算

单位: %

年份	产出增长	劳动投入增长	资本投入增长	全要素生产率增长
	$\Delta Y/Y$	$(1 - \alpha) \Delta L/L$	$\alpha \Delta K/K$	$\Delta A/A$
1978—2015	9.4	2.7	4.7	2.0
1978—1981	4.2	1.8	2.7	-0.3
1982—1986	9.5	2.6	3.6	3.3
1987—1990	4.8	5.3	2.6	-3.1
1991—1999	9.6 ↑	1.6 ↑	3.4 ↓	4.6 ↑
2000—2008	9.5 ↑	1.4 ↑	4.4 ↓	3.7 ↑
2009—2015	7.1 ↓	0.7 ↓	4.9 ↓	1.5 ↓

资料来源:作者根据公开数据计算。

Example 2 (观察与思考)

中国经济增长的动力是什么? 什么是新质生产力?

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Policies to Promote Growth

西方经济学

- (1) Evaluating and changing the rate of saving: $s \leq s_{Gold}$
- (2) Allocating the economy's investment: Human capital, physical capital with spillovers, and public capital.
- (3) Establishing the right institutions.
- (4) Supporting a pro-growth culture: treatment of women; attitudes toward children; the degree of openness to new ideas; and trust.
- (5) Encouraging technological progress.

信任弥补法律的不足.



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ARTICLES

<https://doi.org/10.1038/s41562-021-01062-3>

nature
human behaviour



Skill levels and gains in university STEM education in China, India, Russia and the United States

Prashant Loyalka^{1,2}✉, Ou Lydia Liu³, Guirong Li⁴, Elena Kardanova⁵, Igor Chirikov^{5,6}✉, Shangfeng Hu⁷, Ningning Yu⁸, Liping Ma⁹, Fei Guo¹⁰, Tara Beteille¹¹, Namrata Tognatta¹¹, Lin Gu¹³, Guangming Ling¹³, Denis Federiakin¹⁵, Huan Wang¹⁶, Saurabh Khanna¹⁶, Ashutosh Bhuradia¹⁶, Zhaolei Shi¹⁶ and Yanyan Li⁴

Universities contribute to economic growth and national competitiveness by equipping students with higher-order thinking and academic skills. Despite large investments in university science, technology, engineering and mathematics (STEM) education, little is known about how the skills of STEM undergraduates compare across countries and by institutional selectivity. Here, we provide direct evidence on these issues by collecting and analysing longitudinal data on tens of thousands of computer science and electrical engineering students in China, India, Russia and the United States. We find stark differences in skill levels and gains among countries and by institutional selectivity. Compared with the United States, students in China, India and Russia do not gain critical thinking skills over four years. Furthermore, while students in India and Russia gain academic skills during the first two years, students in China do not. These gaps in skill levels and gains provide insights into the global competitiveness of STEM university students across nations and institutional types.

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- (1) 提高储蓄率对稳态人均收入有何影响？提高人口增长率对稳态人均收入有何影响？
- (2) 人均收入长期增长的动力来自哪里？
- (3) What is conditional convergence?
- (4) 一个变量的增长率，为何不能用变量对时间的导数衡量？
- (5) 在 Solow 模型的稳态下，水平变量、人均变量和有效人均变量的增长率分别是什么？
- (6) Solow 模型的基本方程是什么？
- (7) 稳态的决定因素有哪些？每个因素如何影响稳态有效人均资本？
- (8) 为什么 Solow 模型中会出现动态无效率？
资本存量过高，高于黄金律资本存量。
- (9) 有效人均资本越高的国家，是否越富裕？
实际利率低于持平投资的增长率。
- (10) 稳态附近收敛速度的推导和直觉解释。



马工程教材疑难重点

1 (E2, p.281)

根据马工程教材观点，应当如何评析西方经济学的经济增长理论？

2 (E2, p.284)

根据马工程教材观点，经济增长理论是否可以说明中国提出的创新驱动发展战略？

3 (E2, p.28)

根据马工程教材观点，应当如何看待西方经济学的研究方法？

